NUMERICAL MODELLING OF MIXED FLOWS IN HYDROELECTRIC SCHEMES

Modélisation numérique d'écoulements tantôt en charge tantôt à surface libre dans les installations hydroélectriques

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Les écoulements mixtes, définis comme des écoulements tantôt en charge, tantôt à surface libre, peuvent se présenter dans les galeries et conduites des aménagements hydrauliques, provoquant des fluctuations de pression importantes. Le projet ROEMIX, collaboration entreEDF-CIH et l'Université de Savoie, a pour objectif le développement d'un code de simulation numérique 1D traitant des écoulements mixtes. Cette collaboration a conduit à l'écriture d'un système d'équations unique, représentant aussi bien les écoulements en charge que les écoulements à surface libre, et au développement du code de calcul ROEMIX, résolvant ce système d'équations, en régime transitoire à l'aide d'un schéma numérique de type « volumes finis ».

I ■ DESCRIPTION OF HYDROELECTRIC SCHEMES

There are two main types of hydroelectric schemes:

Water retaining schemes,

Water diversion schemes.

These two types may be combined in the same scheme.

Water retaining schemes consist of a very high dam and an adjoining hydroelectric plant.

Water diversion schemes generally consist of a low dam, or even a single water intake without a dam ("intake from below" or "Tyrolean intake") that diverts water from a waterway into a headrace channel or tunnel with a low gradient of invert.

Let us take the example of a scheme with headrace tunnel: depending on whether this tunnel operates in free flow or under pressure in its downstream section, it leads into a pressurisation chamber[2] or a surge tank[3] which is followed, along the line of the greatest gradient, by a metal pipe (penstock) operating under pressure and supplying the hydroelectric plant located at the foot of the slope. A tailrace channel restores the water turbined in the plant to the waterway.

A variation to this layout consists of constructing an underground plant which is then no longer supplied by a penstock but by a shielded vertical shaft. The water must then be returned to the waterway via a tailrace tunnel protected at its head by a surge tank.

STEPs (Stations for the Transfer of Energy by Pumping) may be considered as a combination of water retaining and diversion schemes integrating a pumping plant, an upstream reservoir and a downstream reservoir.

Fig. 1 shows an example of a hydroelectric scheme.

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2. Pressurisation chamber is a simple storage reservoir where the base of the downstream wall is used for inlet into the penstock.

3. A surge tank is a vertical shaft (for the simplest ones) whose goal is to divide the hydroelectric network into two sections in order to prevent the progagation in the gallery of high fluctuations in pressure related on the starting or the stopping of the turbines of the power plant ("water hammer").

Figure 1: Example of a hydroelectric scheme (Photography: all rights reserved). Background: a water retaining scheme (Serre Ponçon dam); forground: the upper part of a water diversion scheme (Espinasses dam and Durance channel).

II ■ LOCATION OF MIXED FLOWS IN HYDROELECTRIC SCHEMES AND ASSOCIATED PROBLEMS

Mixed flows (partly under pressure and partly free flow, Fig. 2) may exist in the headrace and tailrace tunnels of diversion schemes.

In transient operating conditions, at the point at which the water rejoins the crown of the tunnel, these flows generate pressure fluctuations related to the difference in wave velocity between the free flow and the pressurised flow in the tunnel.

Air pockets may also be created, they then travel along the tunnel ("slug flows", Fig. 3), rise in pressure and are expelled violently via aeration (or de-aeration) pipes, which may then be subject to deterioration to their base.

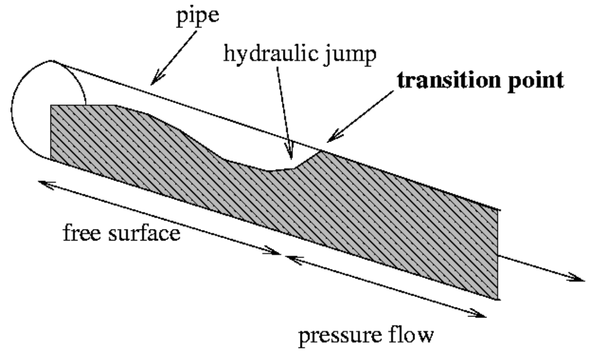


Figure 2: Mixed flow in a pipe.

III ■ NUMERICAL MODELLING OF MIXED FLOWS

● III.1 Roemix numerical code

A computer code capable of simulating transient mixed flows in tunnels would be useful for:

Design calculations for new structures. In this respect reference may be made to:

– The analytical studies of Meyer Peter and Favre (1932) for design calculations of the surge tank of the underground tailrace tunnel with mixed flows at the Wettingen plant (Switzerland),

– The numerical model developed by Wiggert (1972), validated using a test case on a physical model, which resulted in calculation of the pressure levels measured in Wettigen.

Reducing overpressures in tunnels of existing structures, thereby reducing maintenance costs simply by modifying operating instructions (modification of plant charging gradients, management of water intake gates, etc.).

The code in question must be capable of representing a hydroelectric scheme in all its complexity (in addition to the simplified layout described above, it is often necessary to add intermediate inflows, lareral weirs for overflow, air valves and other miscellaneous and varied hydraulic structures).

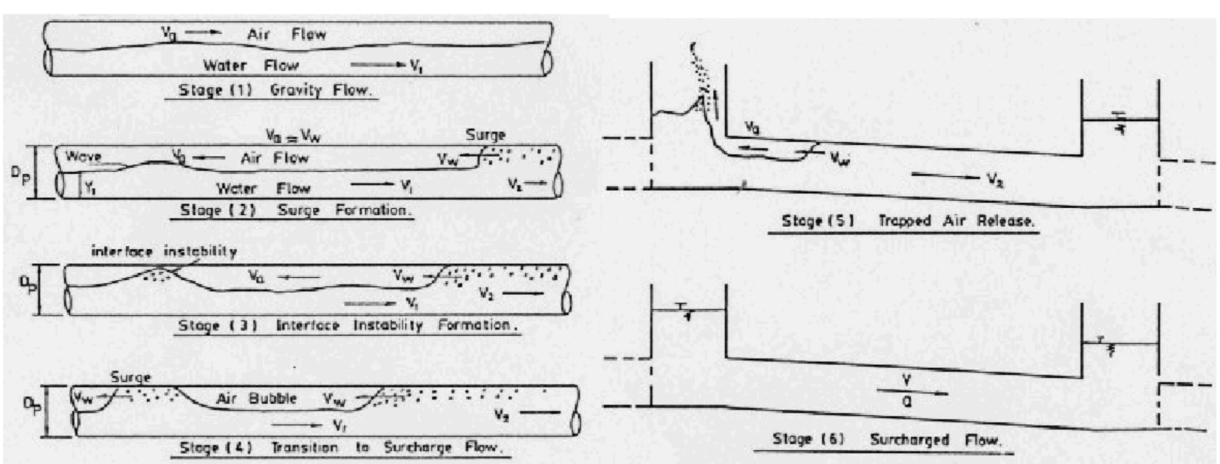


Figure 3: Stages in transition from gravity to surcharge flow in a sewer (after Hamam and McCorquodale 1982).

The most logical development strategy is therefore to develop one transient flow model only on a tunnel section (the ROEMIX model shown below) and then link it to a code for transient pressurised flows that has already been tried and tested in the domain of design calculations for hydroelectric structures (in this case BELIER software (Winckler, 2002)).

The Roemix model (like Wiggert's) does not take account of the effects of air entrained by the flow or possible air pockets trapped during transient operation.

III.2 A conservative model for mixed flows

Before writting our mixed model, let us briefly recall some features about the Saint Venant equations.

III.2.1 Saint-Venant equations revisited

The system of Saint-Venant for flows in an open channel can be written as:

$$\partial_t A + \partial_x Q = 0 \tag{1}$$

$$\partial_t Q + \partial_x \left(\frac{Q^2}{A} + gI_1 \cos\theta(x) \right) = gA(\sin\theta - S_f) + gI_2 \tag{2}$$

The unknowns are the cross-sectional flow area $A = A(x,t)$ and the discharge Q. The other terms are (see Fig. 4) $gI_1 \cos\theta(x)$, term of hydrostatic pressure with $I1 = \int_0^y (h-z)\sigma(x,z)dz$ and gI_2, pressure source term induced by the changes of the geometry, with $I2 = \int_0^h (h-z)\partial_x \sigma(x,z)dz$.

This system can be derived from the incompressible Euler equations by integration over sections orthogonal to the flow axis. The free surface is advected by the flow and is assumed to be horizontal in the *y* direction. The distribution of the pressure is supposed to be hydrostatic: this means that the acceleration of a particle in the plane orthogonal to a streamline is zero. In the case of an uniform geometry of the channel (which may be a pipe, of course) we have $I_2 = 0$ and this is assumed in the sequel. The friction term S_f is assumed to be given by the Manning-Strickler law:

$$S_f = K(A)\bar{u}|\bar{u}| \text{ with } K(A) = \frac{1}{K_s^2 R_h(A)^{4/3}} \tag{3}$$

where $K_s > 0$ is the Strickler coefficient, depending on the material, and $R_h(A)$ is the so called hydraulic radius given by $R_h(A) = \frac{A}{P_m}$, P_m being the wet perimeter (length of the part of the channel's section in contact with the water).

III.2.2 A conservative model for unsteady pressure flows in closed pipes

The commonly used model to describe pressure flows in closed channels is the system of the Allievi equations (Streeter and *al*.,1998). Unfortunately this system of first order partial differential equations cannot be written under a conservative form since this model is derived by neglecting some acceleration terms and then is not adapted to derive an unified formulation for mixed flows. We derived our new conservative model for pressure flows from the 3D system of compressible Euler equations by integration over sections orthogonal to the flow axis. The equation for conservation of mass and the first equation for the conservation of momentum are:

$$\partial_t \rho + div(\rho \vec{U}) = 0 \tag{4}$$

$$\partial_t(\rho u) + div(\rho u \vec{U}) = F_x - \partial_x P \tag{5}$$

with the speed vector $\vec{U} = u\vec{i} + v\vec{j} + w\vec{k} = u\vec{i} + \vec{V}$ where the unit vector $\vec{i}$ is along the main axis (see Fig. 5). ρ is the density of the water. We use the Boussinesq linearised pressure law:

$$P = P_a + \frac{1}{\beta}\left(\frac{\rho}{\rho_0} - 1\right) \tag{6}$$

where ρ_0 is the density under the athmospheric pressure P_a and β the coefficient of compressibility of the water. Exterior strengths $\vec{F}$ are the gravity $\vec{g}$ and the friction $-S_f\vec{i}$. We denote $\theta(x)$ the slope of the pipe at position x. Then equations (4)-(5) become:

$$\partial_t \rho + \partial_x(\rho u) + div_{(y,z)}(\rho \vec{V}) = 0 \tag{6}$$

$$\partial_t(\rho u) + \partial_x(\rho u^2) + div_{(y,z)}(\rho u \vec{V}) = \rho g(\sin\theta - S_f) - \frac{\partial_x \rho}{\beta \rho_0} \tag{7}$$

Assuming that the pipe is infinitely rigid, the equations (7)-(8) are integrated over a cross section $\Omega(x)$. A complete derivation of this model as well as a specific second order numerical treatment are given by Bourdarias and Gerbi (2002).

Using the water proof condition $\vec{U}.\vec{N} = 0$ and some classical approximations we get the following system:

$$\partial_t(\rho A) + \partial_x(\rho Q) = 0 \tag{9}$$

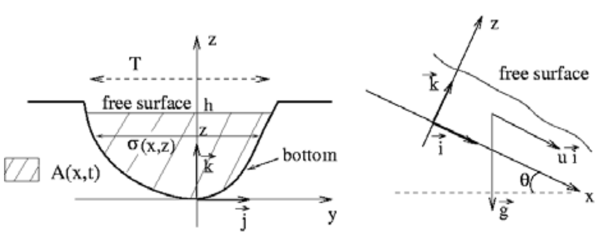


Figure 4: Free surface flow in an open channel.

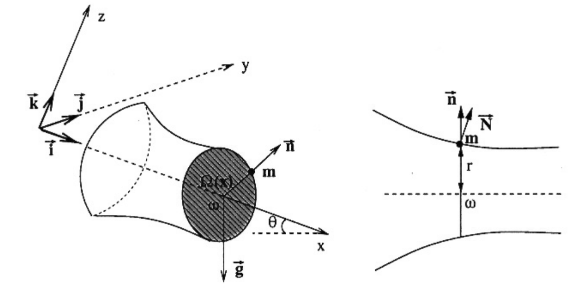


Figure 5: Pressure flow in a pipe.

$$\partial_t(\rho Q) + \partial_x\left(\frac{pQ^2}{A} + c^2\rho A\right) = \rho g A(\sin\theta - S_f) + c^2\rho\frac{dA}{dx} \quad (10)$$

Where S_f is given by (3) and $c^2 = \dfrac{1}{\beta\rho_0}$. The term $\dfrac{dA}{dx}$ is related to the geometry of the pipe and assumed to be zero in the sequel (uniform section).

III.2.3 Mixed model

The two preceding models are written under a conservative form and are formally very closed. The main difference arises from the pressure laws. This proximity leads us to use a single couple of conservative variables. Let us consider first a pressure flow. We define an "FS-equivalent" wet area (FS for Free Surface) A_{eq} through the relation $M = \rho A_{\max} = \rho_0 A_{eq}$, where $A_{\max}$ is the cross sectional area, and a "FS-equivalent discharge" Q_{ed} by $D = \rho Q = \rho_0 Q_{ed}$ or $Q_{ed} = A_{eq}u$. Dividing (9) and (10) by ρ_0 we get:

$$\partial_t A_{eq} + \partial_x Q_{eq} = 0 \quad (11)$$

$$\partial_t Q_{eq} + \partial_x\left(\frac{Q_{eq}^2}{A_{eq}} + c^2 A_{eq}\right) = gA_{eq}\sin\theta - gK\frac{Q_{eq}\left|Q_{eq}\right|}{A_{eq}} \quad (12)$$

In the case of a free surface flow and at a transition point we have obviously $A = A_{eq}$ and $Q = Q_{ed}$. In the sequel $U = (A,Q)$ denotes the state vector for both flow types. The mixed model thus writes:

$$\partial_t A + \partial_x Q = 0 \quad (13)$$

$$\partial_t Q + \partial_x\left(\frac{Q^2}{A} + p(x,A,E)\right) = gA\sin\theta - gK(A,E)\frac{Q|Q|}{A} \quad (14)$$

where E denotes the "state" E of the current point x (free surface: $E = FS$, or pressure: $E = Press$). The mere value of A is neither sufficient to determine the pressure law nor the coefficient K in the friction term, except in the case $A \geq A_{\max}$ (the flow is necessarily pressure). If $A < A_{\max}$ the flow may be free surface or pressure but in depression. In order to ensure the continuity of the pressure (and then of the flux) through a transition point we set, according to (2) and (10):

$$\begin{cases} p(x,A,E) = gI_1(A)\cos\theta(x) \text{ if } A \leq A_{\max} \text{ and } E = FS & (15) \\ p(x,A,E) = gI_1(A_{\max})\cos\theta(x) + c^2(A - A\max) \text{ if } E = Press \end{cases}$$

and also $K(A,E) = \dfrac{1}{K_s^2 R_h(A)^{\frac{4}{3}}}$ if $A \leq A_{\max}$

and $E = FS, K(A,E) = \dfrac{1}{K_s^2 R_h(A_{\max})^{\frac{4}{3}}}$ if $E = Press$

In each zone avoiding a transition point, the system is strictly hyperbolic. The transition points from a type of flow to another are of course unknowns of the problem. Notice that *p(x, A, E)* have a discontinuous derivative with respect to x (gradient of pressure) at each transition point. Such a discontinuity is "severe" in the sense that the magnitude of the eigenvalues of the jacobian matrix of the flux changes drastically through this point. We propose to adapt the VF-Roe scheme due to Gallouet and Masella (1996) in order to treat such a discontinuity and to precise a criterion for tracking the transition point.

III.3 Finite volume discretisation

The main axis of the pipe, with length L, is divided in N meshes,

$M_i = [x_{i-1/2}, x_{i+1/2}]$, $1 \leq i \leq N$ with length h_i. Δt denotes the timestep and we set $t_n = n\Delta t$ The discrete unknowns are $U_i^n = \begin{pmatrix} A_1^n \\ Q_i^n \end{pmatrix} 1 \leq i \leq N, 0 \leq n \leq n_{\max}$. The upstream and downstream boundary states are associated to fictive meshes.

Following Greenberg and Leroux (1996) we use a piecewise constant function to approximate the bottom. Adding the equation $\partial_t Z = 0$ related to the altitude of the bed ($\sin\theta = \partial_x Z$), we note W the conservative variable $(Z,A,Q)^t$ and we get the following system:

$$\partial_t W + \partial_x \Phi(x,W) + gA\partial_x Z = TS(W) \quad (16)$$

with $\Phi(x,W) = (0,Q,Q^2/A + p(x,A))^t$
and $TS(W) = (0,0,-gK(A)Q|Q|/A)^t$.

Such an approximation of the topography introduces a stationary wave for each local Riemann problem at the interfaces $x_{i+1/2}$. Let be W_i^n an approximation of the mean value of W on the mesh $]x_{i-1/2}, x_{i+1/2}[$ at time $t_n = n\Delta t$. Integrating the above equations over $]x_{i-1/2}, x_{i+1/2}[\times [t_n, t_{n+1}[$ we deduce a Finite Volume scheme written as follows:

$$W_i^{n+1} = W_i^n - \frac{\Delta t}{h_i}(\Phi(W_{i+1/2}^*(0^-, W_i, w_{i+1})) - \Phi(W_{i-1/2}^*(0^+, W_{i-1}, W_i))) \quad (17)$$

where $W_{i+1/2}^*(\xi = x/t, W_i, W_{i+1})$ is the exact or approximate solution to the Riemann problem at the interface $x_{i+1/2}$ associated to the left and right state *Wi* and W_{i+1}. Notice that the topography does not appear explicitly in this formulation but contributes to the computation of the numerical flux. Following Gallouet *et al*. (2003) we compute $W_{i+1/2}^*(0^-, W_i, W_{i+1})$ using a linearized Riemann problem. In the case of a transition point present at the interface $x_{i+1/2}$, we propose to use a discontinuous linearized matrix (actually piecewise constant) using the (unknown) states on both sides of the trajectory of the transition point. The precise method will be presented in a forthcoming paper. Concerning the computation of the boundary fluxes, we must give as much scalar boundary conditions as incoming characteristic curves. In case of subcritical flow, the "fictive" exterior vectors state are not completely specified and a special boundary conditions treatment is performed. The method that we use is closely related to those studied by Dubois and Kumbaro (1993). It allows the computation of the boundary states using known values at the same time, so it is naturally implicit. It as to be adapted to the case of a transition point located at the boundary. As pointed out above, we have to track the state of each mesh in terms of flow type. Let us set $E = 1$ for pressure flow and $E = 0$ else. Let be E_i^n the known state of the flow in the mesh with index i at time t_n. After computation of the "pseudo wet area" A_i^{n+1} we use the following criterion to determine E_i^{n+1}:

– if $E_i^n = 0$ then E_i^{n+1} is directly determined by A_i^{n+1}: if $A_i^{n+1} < A_{max}$ then $E_i^{n+1} = 0$, else $E_i^{n+1} = 1$,

– if $E_i^n = 1$ then we postulate that the new state may be free surface only in the case where a transition point crosses the mesh, that is (thanks to an usual CFL restriction on the time step) if at least one adjacent mesh is free surface at time t_n: if $A_i^{n+1} \geq A_{max}$ then $E_i^{n+1} = 1$, else $E_i^n = E_{i-1}^n . E_{i+1}^n$.

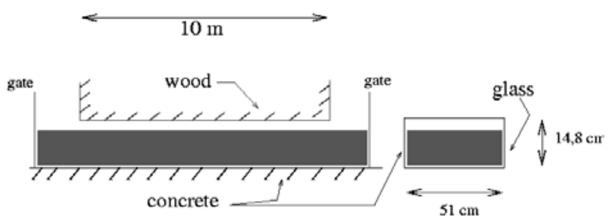
Notice that this procedure allows to distinguish between free surface and depression when $A_i^{n+1} < A_{max}$.

III.4 Numerical validation

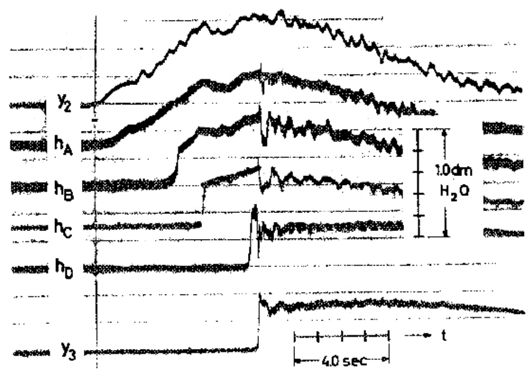
The following test case, is due to Wiggert (1972). The experimental device (see Fig. 6) is an horizontal closed pipe with width 0.51 m and height 0.148 m. The Manning number is $1/K_s^2 = 0.012$. The initial conditions are a stationary state with the discharge $Q_0 = 0$ and the water level $h_0 = 0.128$ m.

A wave coming from the left side causes the pressurization of the closed channel. The upstream condition is a given hydrograph (y_2 in Fig. 6), at the downstream end, a step function is imposed: the water level is kept constant until the wave reachs the exit. At this time, the level is suddenly increased (see $y_3$in Fig. 6).

Fig. 7 gives an example of our results. It concerns the pressure head computed at 3.5 m from the tunnel entrance (solid curve). Circles represent experimental data read on curve H_B, including maxima and minima points of the oscillating parts. We point out that we did not find in other papers, by authors carrying out the same simulation, a convenient numerical reproduction of these oscillations: they do not treat the dynamical aspect of the pressure flow, in particular when using the Preismann slot technique (Wiggert, 1972. Garcia-Navarro *et al*., 1994). On the other hand, we found in M. Fuamba (2002) a similar and interesting approach with a non conservative formulation and another numerical method (characteristics).

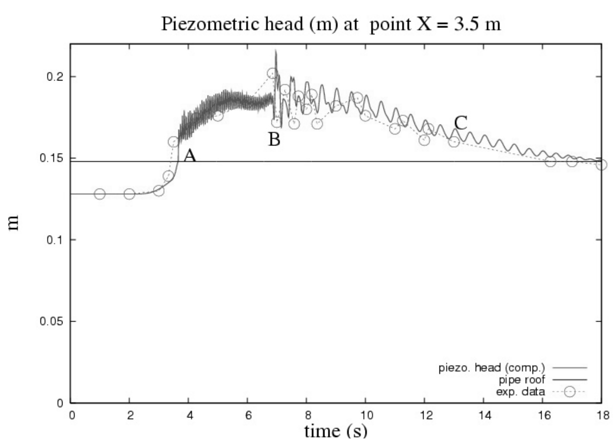


a)



b)

Figure 6: Wiggert experiments (1972) a) Experimental device. b) experimental data. y2: upstream hydrograph, y3: downstream hydrograph. hA, hB, hC, hD: pressure head at 0.5 m, 3.5 m, 5.5 m and 9.5 m from the tunnel entrance (location of recording instruments).



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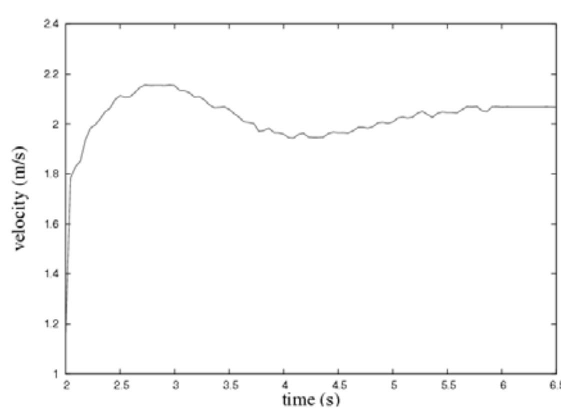

b)

Figure 7: Roemix results a) Piezometric head at location X = 3.5 m (corresponding to hB) with c = 40 m/s. b) velocity of the transition point.

The value of the sound speed c was taken equal to 40 m/s, roughly according to the frequency of the oscillations observed during the phase of total submersion of the tunnel. This low value can be explained by the structure of the tunnel and by bubble flow (see Hamam and McCorquodale (1982) for instance).

We observe that the front reachs the control point at 3.6 s, in a good agreement with the experimental data (less than 0.15 s late). Before it reachs the exit (part AB in Fig. 7a) the oscillations of pressure associated with the moving front reflect between upstream and the front itself (the free surface is at constant pressure) where the channel is flooded. Beyond point B the oscillations result from the step in the downstream water level and they propagate in the fully pressurized flow (their frequency was estimated using the BC part of the experimental curve).

Fig. 7b gives the evolution of the front's speed. We observe the same behaviour as in (Wiggert, 1972): the front quickly attains a maximum speed, decelerates and then slowly accelerate as it approachs the tunnel exit. Moreover the values are consistents with those of Wiggert. Notice that the speed of the front is not very dependent on the value of c.

Our method was implemented in the code "ROEMIX" which is also able to deal with only free surface or only pressure flows (as well as mixed flows). It was also validated in these particular cases situations.

IV ■ CONCLUSIONS

We have described in this paper a new method to simulate mixed flows and the related phenomena using a first order Finite Volume method. It relies on a conservative model describing both free surface and pressure flows and a specific numerical treatment of the transition points. The comparison with experimental data shows a good agreement (speed of the transition point, pressure level).

To be operational in an industrial environment, the Roemix code must be coupled with a code solving transients pressure flows in hydroelectric schemes (BELIER software (Winckler, 2002)).

A possible evolution of the model could be to take into account the effects of air entrained by the flow, like made for example by Hamam and McCorquodale (1982).

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